MOL 410/510: Introduction to Biological Dynamics Fall 2012
Problem Set #6 (due 11/16/2012)
2 Questions, all are **MUST DO**

1. Consider a system where two neurons mutually inhibit each other.

$$\dot{x}_1 = f(x_2) - x_1 \tag{1}$$
$$\dot{x}_2 = f(x_1) - x_2. \tag{2}$$

where x_i represents the membrane potential of neuron i, and

$$f(x_i) = \frac{1}{1 + e^{4\beta(x_i - 0.5)}}, \tag{3}$$

represents its firing rate and is a sigmoidal function between 0 and 1 with slope $-\beta$ at the origin.

Plot the bifurcation diagram for this system as a function of β. (Hint: use the nullclines.) What type of bifurcation happens at $\beta = 1$?

Now let's give these two neurons different stimuli k_1 and k_2

$$\dot{x}_1 = f(x_2 - k_1) - x_1 \tag{4}$$
$$\dot{x}_2 = f(x_1 - k_2) - x_2. \tag{5}$$

where $k_1, k_2 > 0$ represent excitatory stimuli. Explore the dynamics qualitatively through simulation. Why might a group of mutually inhibiting neurons be called a "winner-take-all" network?

2. In class and on pages 245-248 of Kaplan & Glass (available in PDF on the Homework and Solutions page on the course website), we were introduced to the Fitzhugh-Nagumo equations,

$$\frac{dv}{dt} = I - v(v-a)(v-1) - w \tag{6}$$

$$\frac{dw}{dt} = \epsilon(v - \gamma w). \tag{7}$$

These models show a current pulse injected into an axon can generate an action potential. By injecting current steadily, it is possible to generate repeated action potentials. Use the values $\epsilon = 0.008$, $a = 0.139$, and $\gamma = 2.54$.

(a) Use linear stability analysis to figure out how large the current I needs to be to destabilize the fixed point in the model. The sequence of steps you will need to follow is:

i. Find the fixed point as a function of I. It is actually possible to solve the cubic equations algebraically for v. You will need to use the symbolic math toolbox in Matlab (or Mathematica). Read through the Getting started section of the symbolic math toolbox section of the Matlab help menu and also learn how to use the "solve" function. Alternatively, you can find the solution to the cubic in many mathematics handbooks.

ii. Linearize the equations about the fixed point.

iii. Find the eigenvalues of the linear equations. If the real part of all eigenvalues is less than 0 for a given value of I, the fixed point is stable.

(b) Show the trajectory for a value of I where the fixed point is unstable.