MOL 410/510: Introduction to Biological Dynamics Fall 2011
Problem Set #5, Nonlinear dynamics and limit cycles (due 10/26/2012)
1 Question

This "mini-homework" is due after the exam. **However, it is strongly recommended that you do it before the exam, as it pulls together many concepts from recent weeks.** Clearly explain your answers and show your work.

1. **Chemical Reaction**. The following equations describe the chlorine dioxide-iodine-malonic acid (ClO_2 - I_2 - MA) reaction:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = a - x - \frac{4xy}{1+x^2} \tag{1}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = bx\left(1 - \frac{y}{1+x^2}\right), \tag{2}$$

where x is the dimensionless concentration of negative iodine ions and y is the dimensionless concentration of negative chlorine dioxide ions. (This is actually a reduced form of the reaction based on the idea that some reactions occur very slowly and therefore are treated as a constant). Prove that the system has a stable limit cycle in the positive quadrant $x, y \geq 0$ if a and b satisfy certain constraints.